

Chapter 7

Small-Signal Analysis for BJT and FET circuits

Dr. Henry Ohize

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Electronics: Principles, Concepts and Practices

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Intro

Introduction

1. In the previous section, we introduced transistors and discussed their biasing methods for operation in specific regions.
2. We mentioned that BJTs need to be biased in the active region for signal amplification, while FETs require biasing in the saturation region.
3. Now, our focus is on examining the AC signal response of our biased transistor circuit.
4. We will use the term "small-signal analysis" to describe this process.
5. In small-signal analysis, we limit the AC signal to a relatively small percentage of the transistor amplifier's operational range to avoid signal distortion.



Important Metrics in AC Analysis

Important Metrics in AC Analysis

The important metrics in the AC domain for a two-port network¹ are the:

- Input impedance;
- Output impedance;
- Voltage gain;
- Current gain;
- Power gain; and
- Phase shift.

¹We shall give insight into two-port network in the next section.



Input impedance

Input impedance

Input impedance, denoted as Z_i , is a crucial parameter in amplifiers that indicates how much load or resistance the amplifier presents to the signal source when the signal source is connected to the amplifier's input. It is like the "friendliness" of the amplifier towards the signal source.

When we connect a signal source v_s with internal resistance R_s so that a voltage v_i is seen across the amplifier's input, it results in a certain current i_i . Figure 1 shows the circuit for determining the input impedance Z_i , which is given as:

$$Z_i = \frac{|v_i|}{|i_i|}$$



Input impedance continued . . .

If the amplifier's input impedance Z_i is too low (much smaller than the source's internal resistance R_s), it will impact the signal source. This means less voltage will be delivered to the amplifier, resulting in a weaker signal. Hence, a higher input impedance is desirable for effective signal transfer.

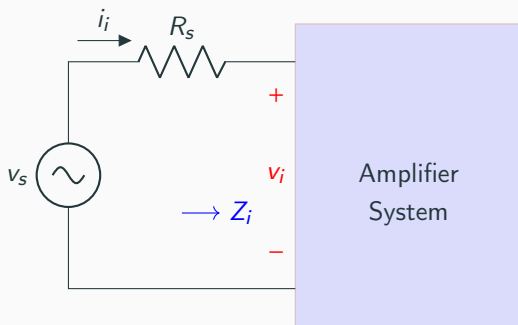


Figure 1: Determination of input impedance

Output Impedance

Output Impedance

Output impedance, denoted as Z_o , refers to the resistance that affects the change in output voltage when the amplifier is providing current to a load. It is measured at the output terminal when examining the system and setting the input signal to zero.

To determine the output impedance in a two-port system, the circuit arrangement of Figure 2 is connected; the input signal is set to zero by our definition and we apply a signal source v to the output half, so that the output current is determined as

$$i_o = \frac{v_o - v}{R_S} \quad (2)$$

We can obtain the output impedance using

$$Z_o = \frac{|v_o - v|}{|i_o|} \quad (3)$$



Output impedance continued ...

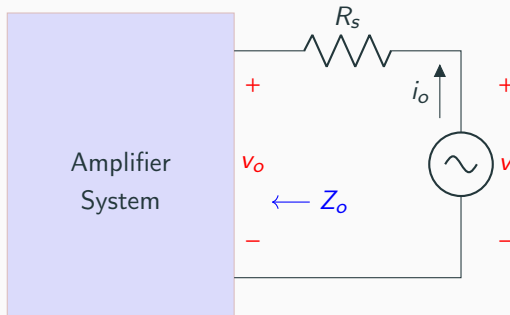


Figure 2: Determination of output impedance

A low output impedance, Z_o is desirable so as to couple a higher proportion of the amplified signal to the load. That is, it is desired that $R_L \gg Z_o$.



Voltage gain

Performance of an amplifier is usually measured in terms of gain, which is the ratio of the output signal to the input signal. When the amplifier is not connected to a load, the gain is referred to as no-load gain and given by

$$A_{VNL} = \frac{v_o}{v_i} \quad (4)$$

If we consider applying a source with resistance R_s to an amplifier as shown in Figure 5, then voltage delivered to the amplifier is

$$v_i = \frac{Z_i v_s}{Z_i + R_s} \quad (5)$$

$$\frac{v_i}{v_s} = \frac{Z_i}{Z_i + R_s} \quad (6)$$



Voltage gain continued . . .

Let us write the signal gain A_{VS} as the ratio of the output signal to the input signal source

$$A_{VS} = \frac{v_o}{v_s} = \frac{v_i}{v_s} \times \frac{v_o}{v_i} \quad (7)$$

Therefore, the signal voltage gain, when we combine Equation (4) and Equation (6), is

$$A_{VS} = \frac{Z_i}{Z_i + R_s} \times A_{VNL} \quad (8)$$

Equation (8) shows that the signal voltage gain A_{VS} is a scaling down of the no-signal voltage gain A_{VNL} by a factor of $\frac{Z_i}{Z_i + R_s}$, thus indicating that a signal source with small internal resistance such that $R_s \ll Z_i$ is desirable for maximum system gain. Also, observe that high input impedance Z_i is desirable for maximum voltage gain.



Current Gain

The amplifier's current gain, denoted as A_i , also known as the current transfer ratio, refers to the ratio of the output current to the input current of an amplifier. It measures the amplification or multiplication factor of the current provided by the amplifier.

$$A_i = \frac{i_o}{i_i} \quad (9)$$

For a loaded amplifier, the current gain is

$$A_i = -A_v \times \frac{Z_o}{R_L} \quad (10)$$

The negative sign is introduced because, in a typical amplifier circuit, the output voltage is 180 degrees out of phase with the output voltage.



Phase shift

Another crucial parameter to consider is the phase shift of the output signal in relation to the input signal. This phase shift refers to the angular difference between the output and input signals and is usually expressed in degrees.

In the context of typical transistor amplifiers, the operational frequencies are often such that we can disregard reactive elements. This simplification means that phase shifts can be either a 180-degree shift or no shift at all (in-phase).

Equation (10) introduces a negative sign, which signifies a 180-degree phase shift. This means that the output signal is flipped by half of a cycle, which corresponds to a 180-degree or π radians phase shift, compared to the input signal.



Two-Port Network

Two-Port Network

The main objective of modeling is to predict how a device will behave when used in a network or circuit. In Chapter 3, we explored different models for diodes. Diodes, along with resistors, inductors, and capacitors, are considered one-port devices. In contrast, a two-port network consists of two pairs or ports of terminals, totaling four terminals or a quadrupole.

Two-Port Network

A two-port network refers to an electrical network or device that has two distinct ports or sets of terminals for connecting to external circuits.

Typical examples of devices that can be modeled as two-port networks include transformers, operational amplifiers, and the transistor, which is our primary focus.



Two-port Network continued . . .

To characterize a two-port network, we establish equations that relate the terminal quantities: input voltage (v_i), output voltage (v_o), input current (i_i), and output current (i_o). Two of these quantities are considered independent variables, while the other two are dependent variables. The parameters that describe these relationships are known as two-port network parameters. Examples of two-port network models or parameters include:

- Impedance parameters (Z -parameters);
- Admittance parameters (Y -parameters);
- Hybrid parameters (H -parameters);
- Inverse hybrid parameters (B -parameters); and
- Transmission parameters.



Hybrid Equivalent Model

Hybrid Equivalent Model

Figure 3 is a two-port network. The hybrid model utilizes hybrid parameters to represent the two-port network.

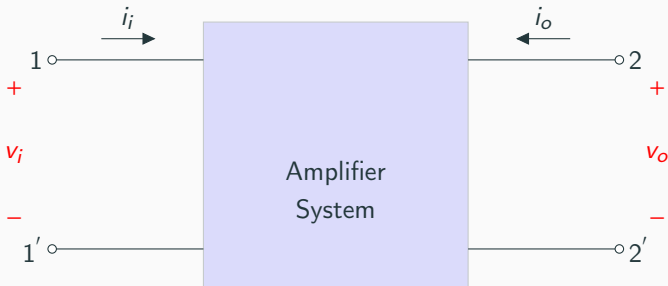


Figure 3: Two-port hybrid network

The hybrid model uses two dependent variables, the input voltage v_i and the output current i_o , along with two independent variables, the input current i_i and the output voltage v_o .



Hybrid Equivalent Model . . .

These variables are related by the following equations:

$$v_i = h_{11} i_i + h_{12} v_o \quad (11)$$

$$i_o = h_{21} i_i + h_{22} v_o \quad (12)$$

Equation (11) represents the input network, while Equation (12) represents the output network. The combination of voltage and current variables in the equations gives rise to the hybrid nature of the parameters, hence the name "hybrid" parameters. To determine the meaning and magnitude of these parameters, we isolate the other terms in the equations one after the other.



Hybrid Equivalent Model ...

With these definitions, we can rewrite Equations (11) and (12) as:

$$v_i = h_i i_i + h_r v_o \quad (13)$$

$$i_o = h_f i_i + h_o v_o \quad (14)$$

We represent the hybrid parameters by drawing the circuit representation of Equations (13) and (14). The combined input and output network is referred to as the hybrid equivalent circuit, as shown in Figure 4.

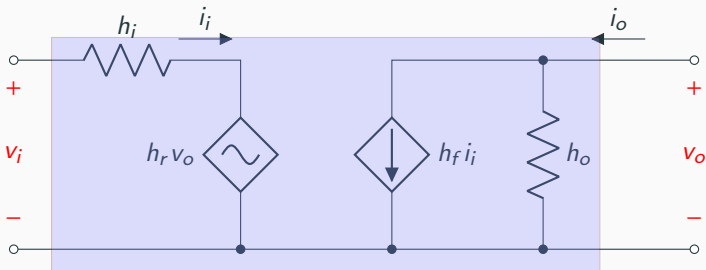
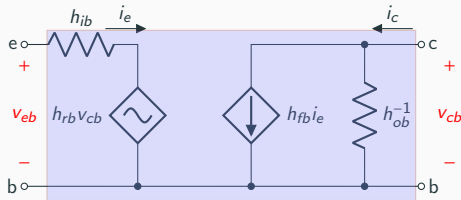


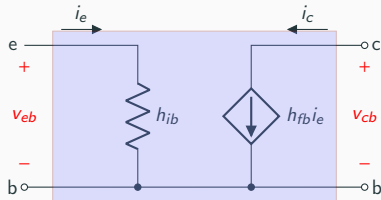
Figure 4: The hybrid equivalent model

Common-base hybrid model

The two-port model in Figure 4 may be applied to the three-terminal transistor. However, a second subscript is added to the h parameters, indicating the common terminal. For the CB connection, the h parameters are denoted as h_{ib} , h_{rb} , h_{fb} , and h_{ob} . You can refer to Figure 5 for a visual representation.



(a) The exact hybrid model for CB



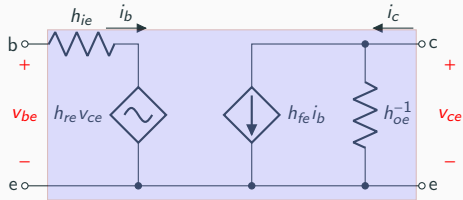
(b) The approximation hybrid model for CB

Figure 5: The approximation and exact hybrid model for CB

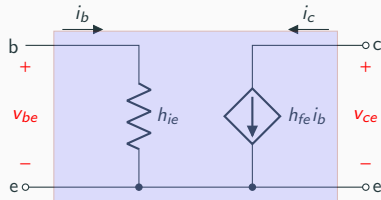


Common-emitter hybrid model

In the CE connection, the h -parameters are denoted as h_{ie} , h_{re} , h_{fe} , and h_{oe} , where the second subscript 'e' represents the common-emitter mode of connection. You can refer to Figure 6 for a visual representation. The input voltage and current are represented as v_{be} and i_b , respectively, while the output voltage and current are denoted as v_{ce} and i_c .



(a) The Exact Hybrid Model for CE



(b) The approximation hybrid Model for CE

Figure 6: The approximation and exact hybrid model for CE



r_e Parameter Transistor Model

r_e Parameter Transistor Model

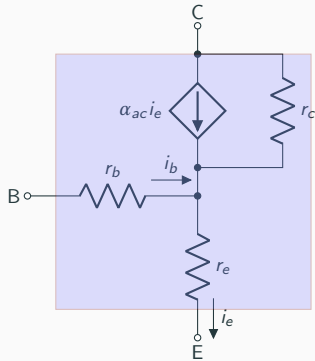
The r_e model is derived from the operating conditions of the transistor and utilizes a diode and a controlled current source to define the transistor's behavior. Figure 7(a) depicts the r_e model, while the simplified approximation is shown in Figure 7(b). The resistance r_e appears between the emitter and base terminals, given as

$$r_e = \frac{26 \text{ mV}}{I_E} \quad (15)$$

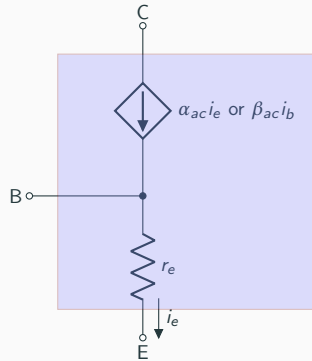
Here, I_E is the DC emitter current obtained from the DC biasing of the transistor.



r_e parameter ...



(a) Generalized r_e parameter model for a BJT



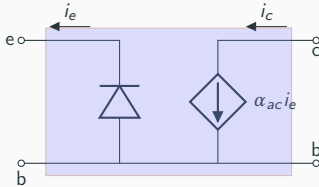
(b) Simplified r_e parameter model for BJT

Figure 7: r_e model for BJT

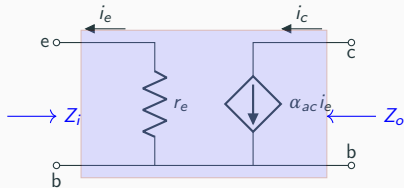


Common-base r_e model

Now, let us incorporate a CB npn transistor into our two-port network, as illustrated in Figure 8. Note that we would not include the usual external circuitry of a CB amplifier at this point. Our objective is to calculate the input and output impedance, voltage gain, current gain, and phase relationship.



(a) r_e model for CB configuration



(b) CB r_e equivalent model for BJT



Figure 8: Two-port network for an npn CB configuration

AC parameters of Common-base r_e model

1. Input impedance for CB r_e transistor model

$$Z_i = r_e = \frac{26 \text{ mV}}{I_E} \Omega \quad (16)$$

Here, I_E refers to the DC biasing current in the emitter terminal.

2. Output impedance for CB r_e transistor model.

$$Z_o \approx \infty \Omega \quad (17)$$

3. Voltage gain for CB r_e transistor model

$$A_v \approx \frac{R_L}{r_e} \quad (18)$$

4. Current gain for CB r_e transistor model

$$A_i = \frac{-i_c}{i_e} = \frac{-\alpha_{ac} i_e}{i_e} = -\alpha_{ac} \quad (19)$$



Common-emitter r_e model

To understand the CE r_e model, let us introduce a CE npn transistor into our two-port network as illustrated in Figure 9. At this point, we shall not include the usual external circuitry of a CE amplifier. Our goal is to analyze the input and output impedance, voltage gain, current gain, and phase relationship, just like we did for the CB configuration.

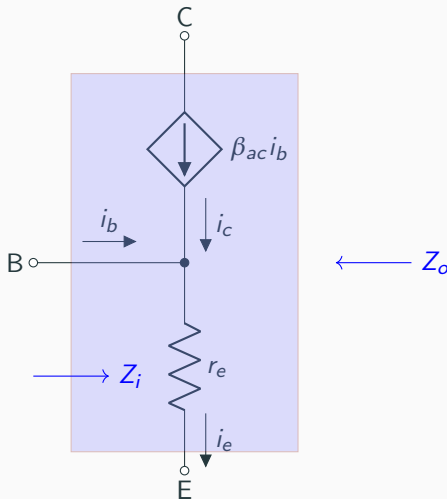


Figure 9: Two-port network equivalent circuit for CE configuration



AC parameters of Common-emitter r_e model

1. Input impedance for CE r_e transistor model

$$Z_i \approx \frac{\beta_{ac} i_b r_e}{i_b} = \beta_{ac} r_e \quad (20)$$

2. Output impedance for CE r_e transistor model.

$$Z_o = r_o \ \Omega \quad (21)$$

3. Voltage gain for CE r_e transistor model

$$A_v = \frac{v_o}{v_i} = -\frac{R_L}{r_e} \quad (22)$$

4. Current gain for CE r_e transistor model

$$A_i = \frac{i_o}{i_i} = \frac{i_c}{i_b} = \frac{\beta_{ac} i_b}{i_b} = \beta_{ac} \quad (23)$$

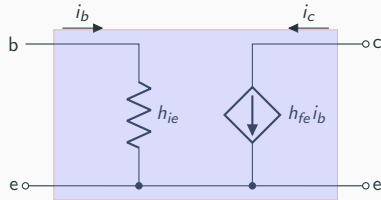


Similarity of the hybrid and r_e model

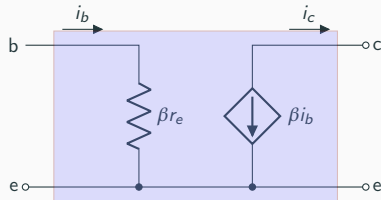
By examining Figures 10(a) and 10(b), we can observe the following relationships:

$$h_{ie} = \beta r_e \quad (24)$$

$$h_{fe} = \beta \quad (25)$$



(a) The approximation hybrid model for CE configuration



(b) r_e model for CE configuration



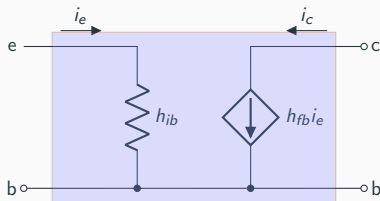
Figure 10: Hybrid versus r_e model for CE

Similarity of the hybrid and r_e model

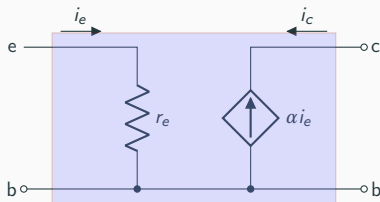
By examining Figures 11(a) and 11(b), we can observe the following relationships:

$$h_{ib} = r_e \quad (26)$$

$$h_{fb} = -\alpha \approx -1 \quad (27)$$



(a) The approximation hybrid model for CB configuration



(b) r_e model for CB configuration



Figure 11: Hybrid versus r_e model for CB

Recommendation:

See worked examples in reference material!

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BJT: AC Signal Modelling

BJT: AC Signal Modelling

The following steps are involved in setting up AC analysis:

1. Set all the DC voltage sources to zero.
2. Replace capacitors with short circuits: Capacitors used for coupling or decoupling ac signals becomes short circuits at the frequency of operation.
3. Remove elements bypassed by short circuits: Due to the short circuits introduced in steps 1 and 2, certain elements may become bypassed or no longer participate in the AC analysis. These elements can be removed from the circuit.
4. Replace the transistor with an appropriate AC equivalent model: To simplify the circuit for AC analysis, we replace the transistor with an equivalent model that captures its AC behavior.
5. Finally, we redraw the circuit in a more convenient and simplified manner, reflecting the changes made in the previous steps.



FET: AC Signal Modelling

FET: AC Signal Modelling

The important metrics in the AC domain for FET amplifier, similar to BJT amplifiers, are voltage gain, input impedance, output impedance and phase shift. However, due to the very high input impedance of FETs, the input current is generally assumed to be zero, making the current gain undefined.

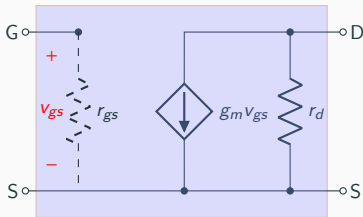
Figure 12(a) represents the complete model, specifically for the common-source (CS) configuration. The resistance r_{gs} is an internal parameter and can be replaced by an open circuit (see Figure 12(b)), as it is typically large.

A change in gate voltage (v_{gs}) controls the level of drain current (i_d), meaning $i_d \propto v_{gs}$. The constant of proportionality for the FET device is known as the transconductance (g_m).

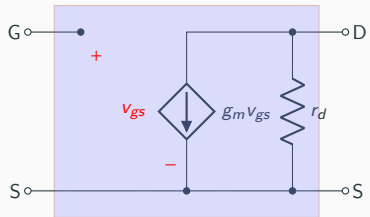


FET: AC Signal Modelling

It is important to note that the equation relating g_m dictates the type of FET being used, such as JFET, DMOSFET, or EMOSFET.



(a) Complete FET CS equivalent model



(b) Simplified FET CS equivalent model

Figure 12: FET common-source AC equivalent circuits



Transconductance parameter

The transconductance can be determined from the slope of the transfer characteristics curve, as shown in Figure 13, or obtained from the datasheet.

The transconductance expression is

$$g_m = g_{m0} \left[1 - \frac{V_{GS}}{V_P} \right] \quad (28)$$

$$= g_{m0} \sqrt{\frac{I_D}{I_{DSS}}} \quad (29)$$

Equation 28 applies to both JFET and DMOSFET devices, but not EMOSFETs.

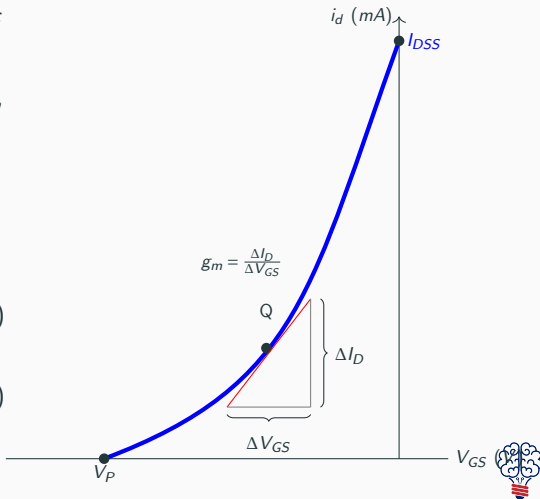


Figure 13: Definition of g_m using transfer characteristic

Recommendation:

See worked examples in reference material!

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