

Chapter 9

Amplifier Frequency Response

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Electronics: Principles, Concepts and Practices

Table of contents

1. Introduction
2. Frequency Response Fundamentals
3. Review of First-Order Filters
4. Frequency Response for Combined Network Elements
5. Frequency Response of BJT and FET Amplifiers



Intro

Introduction

1. Previously, we focused on various aspects of amplifiers without considering signal frequencies.
2. Assumptions in previous discussions was that the amplifiers were in the mid-band range. So that
 - Capacitive elements (coupling and bypass capacitors) acted as short circuits.
 - Internal transistor capacitors acted as open circuits.
3. Outside the mid-range frequency, capacitive reactances start to have an effect.
4. In this chapter, we explore the relationship between voltage gain, phase shift, and frequency.
5. Importance of understanding this relationship include:
 - Crucial for designing amplifiers that provide uniform amplification across all signal frequencies within the signal spectrum of interest.
 - Helps avoid distortion.



Frequency Response Fundamentals

Decibel

The decibel (dB) is a unit used to express the ratio between the output and input of field quantities like voltage or power. It serves as an auxiliary unit and is dimensionless since it represents a ratio of identical quantities. However, when a fixed reference value is used, the decibel can also function as a pseudo unit or level, such as the decibel Watt (dBW). The decibel is commonly used to measure the gain of active devices like amplifiers or the loss in passive devices acting as attenuators, such as the length of a cable.

The power gain in decibels can be calculated using the formula:

$$A_{\text{PdB}} = 10 \log_{10} \frac{P_2}{P_1} \quad (1)$$

The voltage gain in decibels can be calculated using the formula:

$$A_{\text{VdB}} = 20 \log_{10} \frac{V_2}{V_1} \quad (2)$$



Comparing the Decibel Gain and the Linear Gain

In Table 1, the effects of doubling and halving the ordinary voltage or power gain are shown on the decibel scale.

Table 1: Decibel Gain VS Voltage VS Power Levels

dB Gain	Voltage Gain A_V	Power Gain A_P
-6	0.5 (halving)	0.25
-3	0.7071 or $\frac{1}{\sqrt{2}}$	0.5 (halving)
0	1	1
3	1.414 or $\sqrt{2}$	2 (doubling)
6	2 (doubling)	4
10	3.2	10
20	10	100



Signal representation in dBW, dBm and dBV

1. The decibel scale is not only useful for measuring gain ratios, but can also be employed to express power and voltage levels in a logarithmic form.
2. For power measurements, we can select a reference of 1 Watt, denoted as dBW, or 1 milliwatt, denoted as dBm. Mathematically, this can be expressed as follows:

$$P_{\text{dBW}} = 10 \log_{10} \frac{P}{1 \text{ Watt}} \quad (\text{dBW}) \quad (3)$$

$$P_{\text{dBm}} = 10 \log_{10} \frac{P}{1 \text{ mWatt}} \quad (\text{dBm}) \quad (4)$$

3. Similarly, for voltage measurements, we can adopt a similar approach. For example, selecting a reference of 1 Volt would yield a unit of dBV. In general, the equation for voltage measurement can be written as:

$$V_{\text{dBV}} = 20 \log_{10} \frac{V}{\text{reference}} \quad (\text{dB (refvolt)}) \quad (5)$$



Cutoff frequency and Bandwidth

- The cutoff frequency is the point where the magnitude (amplitude) response of a filter or system drops to -3 dB below its maximum or nominal value. This corresponds to a reduction in magnitude to approximately 70.7% of the maximum value. The cutoff frequency is usually defined as the point where the reactive components of the inductor X_L or capacitor X_C of a given network equal the resistive component R .
- Bandwidth defines the width of the frequency band within which the filter or system operates linearly, typically allowing signals to pass through with minimal attenuation. Usually, the difference between the upper and lower cutoff frequencies is the bandwidth. Mathematically, the bandwidth is calculated as:

$$BW = f_H - f_L$$



Octave and Decade

- An octave in electronics refers to a frequency ratio. When you double a frequency, you go up one octave, and when you halve a frequency, you go down one octave.

To put it simply, if your frequency is 100 Hz, then 200 Hz is one octave higher. How many octaves higher is 800 Hz compared to 100 Hz? It is three octaves higher.

- A decade represents a tenfold change in frequency.

To put it simply, if your frequency is 10 Hz, then a span from 10 Hz to 100 Hz covers one decade, and a span from 10 Hz to 1000 Hz covers two decades. In simpler terms, frequencies of 100 Hz and 1000 Hz each span one and two decades, respectively, compared to a frequency of 10 Hz.



Review of First-Order Filters

Low-pass (LP) filters or lag network

The low-pass filter, also referred to as a lag network, gets its name because the output voltage lags behind the input voltage. This type of filter is represented by the circuit shown in Figure 1.

The key characteristic of the capacitor's reactance is that it is inversely proportional to the frequency of the signal, that is, $X_C \propto 1/\omega$. As the frequency increases, the reactance decreases, allowing higher frequency components to pass through more easily, hence less voltage drop across the capacitor at high-frequency. Conversely, at lower frequencies, the reactance increases resulting in higher voltage drop across the capacitor.

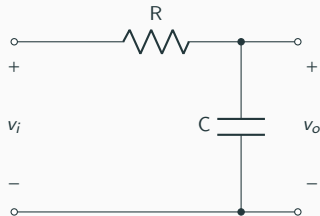


Figure 1: Low-pass filter



The voltage gain for the lag network in Fig. 1, is expressed as

$$A_v = \frac{v_o}{v_i} = \frac{1}{1 + \frac{R}{X_C}} \quad (7)$$

The voltage gain in terms of the cut-off frequency is given as

$$A_v(jf) = \frac{1}{1 + j \frac{f}{f_{cu}}} \quad (8)$$

To represent this complex function in terms of magnitude and phase, we have:

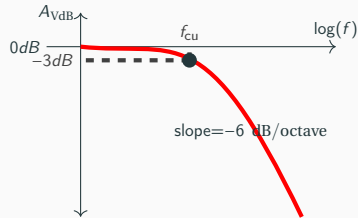
$$A_v(jf) = |A_v| \angle \phi = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{cu}}\right)^2}} \angle -\tan^{-1}\left(\frac{f}{f_{cu}}\right) \quad (9)$$

Equation (9) represents the response of a low-pass filter and can be plotted in both magnitude and phase against frequency. To express the gain in Equation (9) in dB, we use:

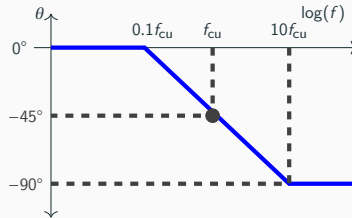
$$A_{VdB} = -10 \log_{10} \left[1 + \left(\frac{f}{f_{cu}} \right)^2 \right] \quad (10) \quad \text{💡}$$

Frequency response of low-pass filter (lag network)

The gain plot of the lag network has a negative slope after the critical or cutoff frequency, f_{cu} , with a decrease of 6 dB per octave or 20 dB per decade. It is important to note the negative slope. The typical gain and phase plots are depicted in Figure 2, which are obtained from Equation (9).



(a) Lag network gain plot



(b) Lag network phase plot



Figure 2: Lag network response

High-pass (HP) filters or lead network

The implementation of the high-pass (HP) filter circuit is depicted in Figure 3. This type of filter is commonly referred to as a lead network. At high frequencies, the reactance $X_C = \frac{1}{\omega C}$ becomes very small, and the output voltage is primarily determined by the resistance across the resistor R , resulting in an essentially resistive output.

We can express the voltage gain relationship as follows:

$$A_v = \frac{v_o}{v_i} = \frac{R}{R + X_C} \quad (11)$$

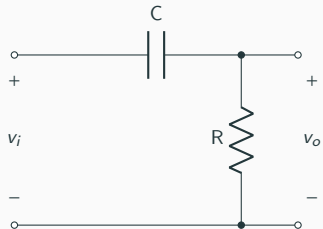


Figure 3: High-pass filter



High-pass (HP) filters or lead network continued ...

We can write the transfer function for the lead network as:

$$A_v(j\omega) = \frac{1}{1 + \frac{1}{j2\pi fCR}} \quad (12)$$

Again, we define the frequency f_{cu} at resonance or cutoff frequency as the frequency where the network resistor R is equal to the capacitive reactance X_C , so that,

$$R = \frac{1}{2\pi f_{cu}C} \quad (13)$$

Substituting Equation (13) into Equation (12)

$$A_v(jf) = \frac{1}{1 - j \frac{f_{cu}}{f}} \quad (14)$$



High-pass (HP) filters or lead network continued ...

Equation (14) may be expressed in magnitude and phase as:

$$A_v(jf) = |A_v| \angle \phi = \frac{1}{\sqrt{1 + \left(\frac{f_{cu}}{f}\right)^2}} \angle \tan^{-1} \left(\frac{f_{cu}}{f} \right) \quad (15)$$

Equation (15) can also be expressed in decibels (dB) as:

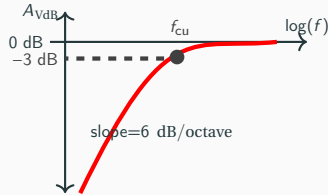
$$A_{v\text{dB}} = 20 \log_{10} |A_v| \quad (16)$$

$$A_{v\text{dB}} = -10 \log_{10} \left[1 + \left(\frac{f_{cu}}{f} \right)^2 \right] \quad (17)$$

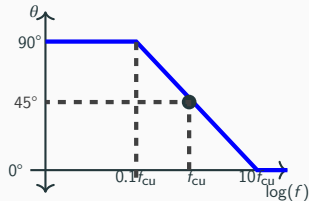


Frequency response of high-pass filter (lead network)

In a passive network, a decrease in voltage gain at low frequencies is caused by a lead (HP) network. At high frequencies, the circuit behaves mainly resistively, with the output voltage (v_o) being approximately equal to the input voltage (v_i). The gain and phase plots of the lead network are shown in Figure 4.



(a) Lead network gain plot



(b) Lead network phase plot



Figure 4: Lead network frequency response

Bode plot overview

The frequency response plots of the Lead and Lag network discussed so far are known as Bode plots. A Bode plot is a graphical representation of the frequency response of a system, and it is widely used for circuit design and analysis. It consists of two plots: one shows the normalized gain of the system in decibels (dB) against frequency, and the other displays the phase shift of the signal against frequency. The frequency axis is logarithmically scaled to accommodate a wide range of frequencies.

Figure 5 illustrates a typical Bode gain plot for an RC-coupled amplifier. The plot shows the normalized gain against frequency. The maximum gain occurs within the frequency range between the upper cutoff frequency (f_H) and the lower cutoff frequency (f_L), known as the mid-range. Any gain value below the mid-range or reference level of 0 dB is expressed as a negative dB value. The critical frequencies f_L and f_H are determined by the lead and lag networks formed by the amplifier, respectively.



Bode plot overview continued ...



Figure 5: A typical Gain versus Frequency or Bode gain plot



Frequency Response for Combined Network Elements

Frequency Response for Combined Network Elements

Amplifiers often contain multiple reactive elements, and the overall frequency response is a combination of these individual elements. This section explains how the reactive elements combine to give an overall frequency response for a given amplifier circuit.

The complete frequency response of an amplifier is obtained by summing the frequency responses of the individual networks. In an amplifier circuit, we first need to determine the dominant lead and lag networks. The dominant lead network has the highest low cutoff frequency (f_L), while the dominant lag network has the lowest high cutoff frequency.

To visualize the frequency response using a Bode plot approximation technique, follow these steps:

- Identify the different networks formed by the passive RCL elements.
- Locate the cutoff frequencies (f_{cu}) of the separate networks on the frequency axis.



Frequency Response for Combined Network . . .

- Identify the lead and lag networks along with their cutoff frequencies.
- Note the dominant lead and lag cutoff frequencies.
- Draw a straight line between the cutoff frequencies of the dominant lag and dominant lead networks to represent the mid-band range.
- If there is no lead network (as in DC coupled amplifiers), extend the mid-band gain down to DC (0 Hz).
- Draw a 6 dB per octave slope between the dominant lead frequency and the immediate "low" lead frequency.
- Since the effects of the networks are cumulative, draw a 12 dB per octave slope between the second lead network's cutoff frequency and the third lead network. Thereafter, increase the slope by 6 dB per octave for each subsequent lead network's cutoff frequency.
- Similarly, draw a -6 dB per octave slope between the dominant lag frequency and the immediate "high" lag frequency. Again, increase the slope by -6 dB per octave for each additional lag network's cutoff frequency, considering the cumulative effects of multi-stage amplification.



Recommendation:

See worked examples in reference material!

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Frequency Response of BJT and FET Amplifiers

Frequency response of BJT amplifier



Frequency response of FET amplifier

